§15.5

$$\iiint_D f(x,y,z)\, dv$$

$$\text{volume of } D = \iiint_D dv$$

$$dv = \left.\begin{matrix} dx\,dy\,dz \\ dx\,dz\,dy \end{matrix}\right\}$$

$$\left\{\begin{matrix} dy\,dx\,dz \\ dy\,dz\,dx \end{matrix}\right.$$

$$\left\{\begin{matrix} dz\,dx\,dy \\ dz\,dy\,dx \end{matrix}\right.$$

example 1

D is the region between $z=x^2+3y^2$ & $z=8-x^2-y^2$

$$V=\int_{-2}^{2}\int_{-\sqrt{2-\frac{x^2}{2}}}^{\sqrt{2-\frac{x^2}{2}}}\int_{z=x^2+3y^2}^{z=8-x^2-y^2} dz\,dy\,dx$$

$$=\int_{-\sqrt{2}}^{\sqrt{2}}\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}}\int_{x^2+3y^2}^{8-x^2-y^2} dz\,dx\,dy$$

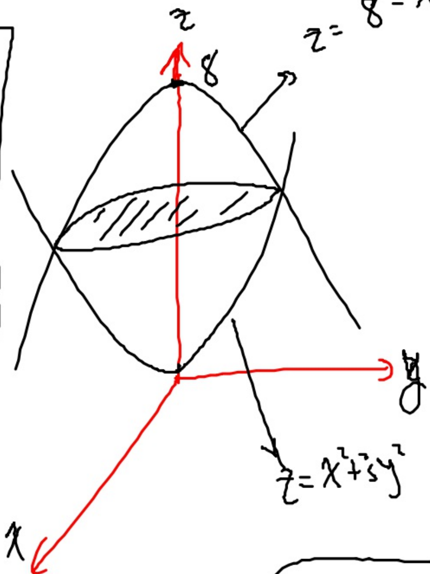
intersection of surfaces

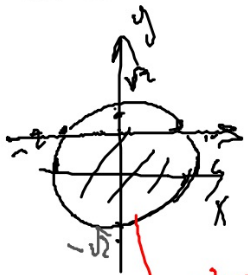
$x^2+3y^2=8-x^2-y^2$

$2x^2+4y^2=8$

$x^2+2y^2=4$

$\frac{x^2}{4}+\frac{y^2}{2}=1$





$x^2+2y^2=4$

$y^2=2-\frac{x^2}{2}$

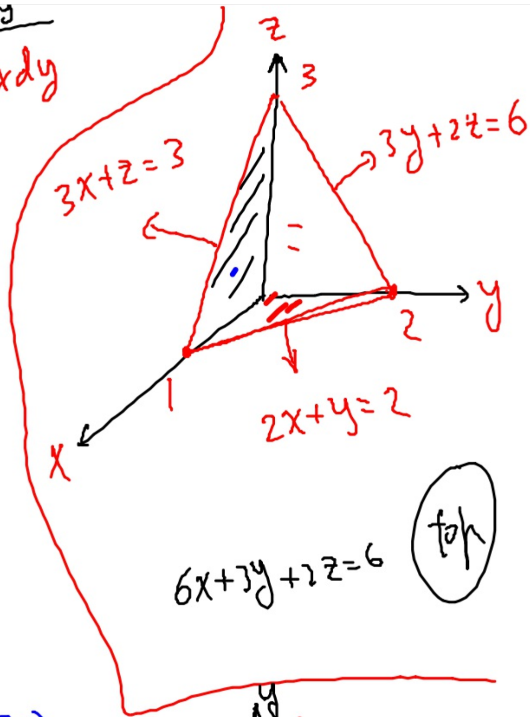
see p 863 for evaluation

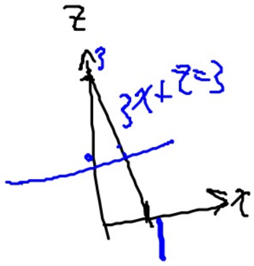
ex #3

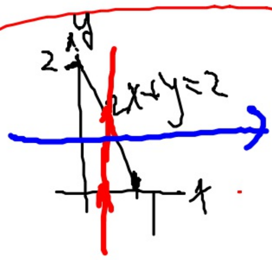
$$V = \int_0^1 \int_0^{2-2x} \int_0^{\frac{6-6x-3y}{2}} dz\,dy\,dx = \int_0^2 \int_0^{\frac{2-y}{2}} \int_0^{\frac{6-6x-3y}{2}} dz\,dx\,dy$$

$$= \int_0^3 \int_0^{\frac{3-z}{3}} \int_0^{\frac{6-6x-2z}{3}} dy\,dx\,dz = \int_0^1 \int_0^{3-3x} \int_0^{\frac{6-6x-2z}{3}} dy\,dz\,dx$$

$$= \int_0^3 \int_0^{\frac{6-2z}{3}} \int_0^{\frac{6-3y-2z}{6}} dx\,dy\,dz = \int_0^2 \int_0^{\frac{6-3y}{2}} \int_0^{\frac{6-3y-2z}{6}} dx\,dz\,dy$$







16) $\int_0^1 \int_0^{1-x^2} \int_3^{4-x^2-y} x \, dz \, dy \, dx$

$$= \int_0^1 \int_0^{1-x^2} x\left[1-x^2-y\right] dy\, dx = \int_0^1 \int_0^{1-x^2} \left(-x^3+x-xy\right) dy\, dx$$

$$= \int_0^1 \left(\left(-x^3+x\right)\left(y\right)\Big|_0^{1-x^2} - x\left[\frac{y^2}{2}\right]_0^{1-x^2} \right) dx = \cdots\cdots\cdots = \frac{1}{12}.$$

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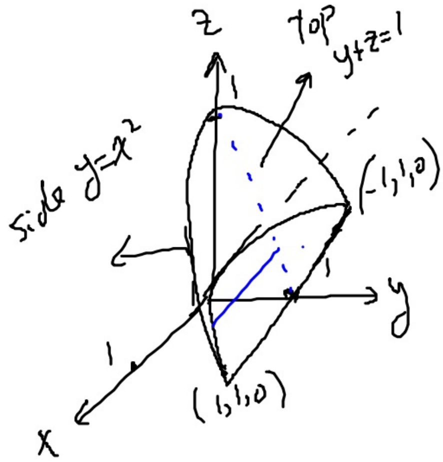
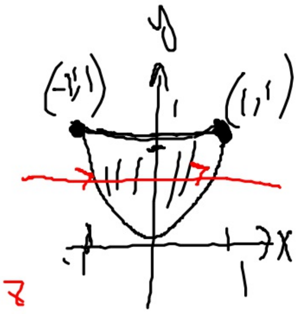
$$V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx = \int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{1-y} dz\,dx\,dy$$

$$= \int_{-1}^{1}\int_{0}^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx = \int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$$

$$= \int_{0}^{1}\int_{0}^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy\,dz = \int_{0}^{1}\int_{0}^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} \cancel{dx\,dz\,dy}$$

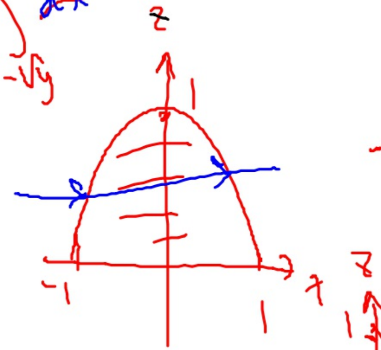




$y = x^2$

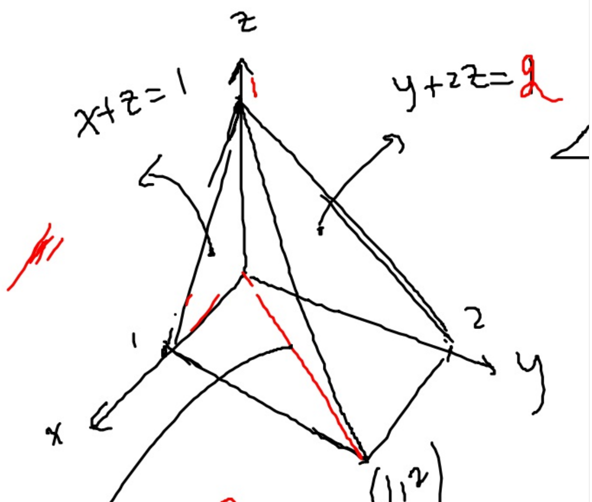
$y = 1 - z$

$\Rightarrow z = 1 - x^2$



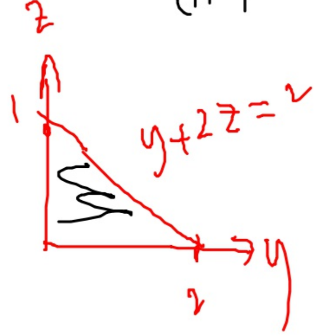
2(4)

$$v = \int_0^1 \int_0^{1-x} \int_0^{2-2z} dy\, dz\, dx$$



or

$$\int_0^1 \int_0^{2-2z} \int_0^{1-z} dx\, dy\, dz$$



$$= \cdots\cdots = \frac{2}{3}.$$

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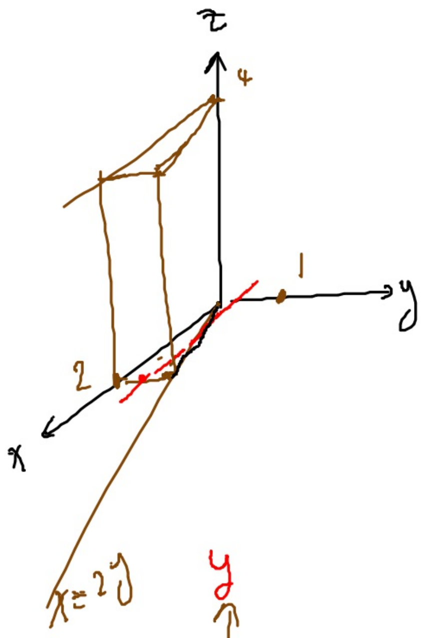
$$\int_0^4 \boxed{\int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dx\, dy}\, dz$$

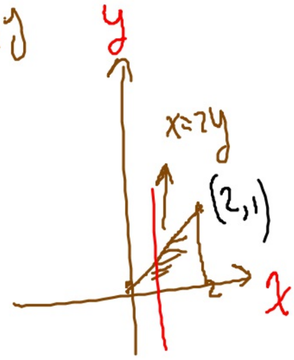
$$= \int_0^4 \boxed{\int_0^2 \int_0^{\frac{x}{2}} \frac{4\cos(x^2)}{2\sqrt{z}}\, dy\, dx}\, dz$$

$$= \int_0^4 \int_0^2 \frac{\cancel{4}\cos(x^2)}{2\sqrt{z}}\, 2\frac{x}{\cancel{2}}\, dx\, dz$$

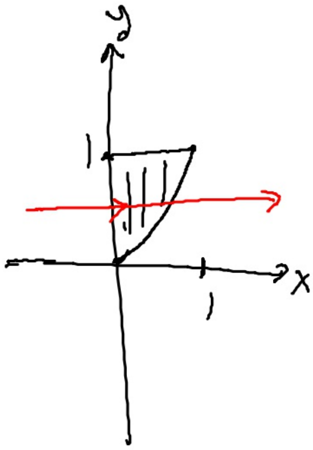
$$= \int_0^4 \frac{1}{2\sqrt{z}} \left(\sin(x^2)\right)_0^2 dz = (\sin 4)\left(\sqrt{z}\right)_0^4$$

$$= 2\sin(4).$$





(42) $\displaystyle\int_0^1\int_0^1\int_{x^2}^1 12xz\, e^{zy^2}\, dy\, dx\, dz$



$$= \int_0^1\int_0^1\int_0^{\sqrt{y}} 12xz\, e^{zy^2}\, dx\, dy\, dz$$

$$= \int_0^1\int_0^1 6yz\, e^{zy^2}\, dy\, dz = 3\int_0^1 \left[e^{zy^2}\right]_0^1 dz$$

$$= 3\int_0^1 \left(e^{z} - 1\right) dz = 3\left(e^{z} - z\right)\Big|_0^1 = \cdots = 3\left(e - 1 - 1\right) = 3e - 6$$